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TEMPERATURE FLEXURE OF ELASTIC ELEMENTS

By G. A. Slomyanskii

Translated from "Some Questions Relating to Modern Instrumentation Technology." Edited by Poliakov.

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TEMPERATURE FLEXURE OF ELASTIC ELEMENTS*

By G. A. Slomyanskii

Various kinds of elastic elements - coil and spiral springs, bellows, aneroid and manometric boxes and units, manometer tubes, force bellows - are widely used in instrumentation and automation as sensitive elements. They are used to measure forces, moments and pressures. In many cases they are employed for indirect measurement of various physical quantities, to each value of which a definite force, moment or pressure corresponds. Thus, for example, elastic members are used to measure the flight altitude and air speed of an airplane, the Mach number, etc.

In many cases instruments and automatic apparatus with elastic elements are intended to operate under ambient temperatures that vary widely. Temperature changes induce changes in the modulus of elasticity of the material of which the elastic element is made, and consequently, variations in its flexure as well, which in turn leads to variation in the readings of the instrument although the quantity being measured has not changed. As a result, a temperature error appears in the instrument readings. In order to calculate the magnitude of this error, we must know the temperature flexure of the elastic element. The temperature flexure of an elastic element depends not only on the temperature but also on the characteristic curve of the element**.

Elastic elements prepared by the same plants and by the identical methods do, as a rule, have slightly differing characteristic curves. Accordingly, they will have slightly different temperature flexures. Hence, in determining the temperature flexure of any specific elastic element, we must take as our basis not its calculated characteristic curve but its actual, experimentally ascertained characteristic curve. This approach to determining the temperature flexure of an elastic element will enable us to solve the problem as to the possibility of using the element in question in an instrument or in some other piece of apparatus for which it is designed.

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**The characteristic curve of an elastic element is the dependence of its course (flexure) on a magnitude of the force acting on it (pressure, force or moment) or any other magnitude for the indirect measurement of which it is used. The characteristic curve may be given either graphically or in tabular form.

The purpose of this article is to work out a method for determining the temperature flexure of an elastic element according to its experimentally determined characteristic curve, taken at a known temperature.

Let us assume that the elastic element is for the purpose of sensing pressures.

We denote: p - pressure acting on the elastic element; W - the give (flexure) of the elastic element, obtained at pressure p. The dependence of the give on the pressure (force or moment) is

$$W = f(p)$$

and this will be called the characteristic curve of an elastic element under pressure (force or moment). We shall assume that the characteristic curve

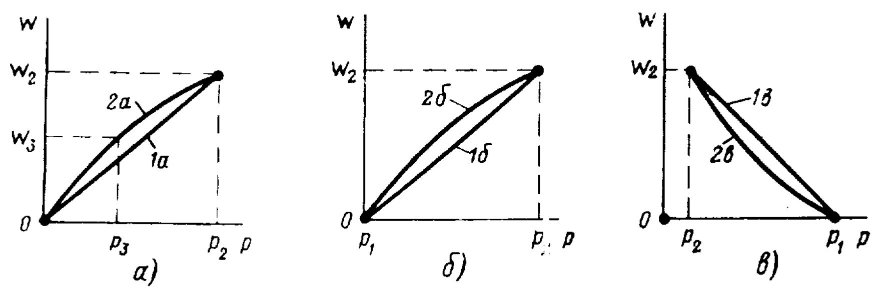

Fig. 1. The drawings of characteristic curves of elastic elements according to pressure.

of the elastic element according to pressure,(force or moment) has no breaks and is a monotonously increasing function in the strict sense (Fig. 1, a and b) or a function monotonously decreasing in the strict sense (Fig. 1, B).

Elastic elements with characteristic curves of this nature are most common. Most frequently the characteristic curves are determined over an interval from 0 to a value p_2 (see Fig. 1, a). However, there are frequent cases in which the characteristic curve does not begin at the 0, but covers a given bounded interval of pressures $/p_1, p_2/$ where p_1 and p_2 are the values for the pressures at the ends of the interval (Fig. 1, b and B). In both cases the characteristic curves may be either linear (Fig. 1, curves 1a and 1b), or nonlinear (Fig. 1, curves 2a, 2b and 2B). The nonlinear curves are most frequently encountered in the case of elastic elements used for indirect measurements. When any elastic element is prepared, it is tested to determine its characteristic curve according to pressure or according to the physical quantity which it is designed to measure indirectly. The dependence between the indirect measurement of the quantity and the pressure is always known. It can usually be given in the form of the corresponding table. In this way, the characteristic curve for pressure is always known for each elastic element that has been made. We now consider how, using this characteristic curve, taken at a given temperature, we can define the magnitude of the temperature flexures of the given elastic element.

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Study of the characteristic curves of elastic elements has shown that the relationship between p and W may correspond, with an accuracy sufficient for practical purposes, to an exponential function, and in its general form, have the characteristic curves shown in Fig. 1a) may be represented by the expression

$$p = aEW^q,$$

where a, q - are constant parameters, dependent on the geometrical dimensions of the elastic element and their Poisson coefficient;
E - the modulus of elasticity of the material of the elastic element.

Solving this equation for W and introducing new constant parameters c and n, we have

$$W = Ap^n \quad A = \frac{c}{E^n} \tag{1a}$$

To determine constants A and n, we must know any two points on the characteristic curve. Let these be, for example, a middle point (p_3, W_3) and a terminal point (p_2, W_2) of the characteristic curve. Then

$$n = \frac{\lg W_2 - \lg W_3}{\lg p_2 - \lg p_3}; \quad A = \frac{W_2}{p_2^n}. \tag{2}$$

Similarly, for the characteristic curves shown in Fig. 1, b and B,

$$W = A(p^n - p_1^n); \tag{1b}$$

$$W = A(p_1^n - p^n); \tag{1B}$$

$$\left(A = \frac{c}{E^n}\right).$$

In order to determine the constants n and A appearing in Eq. (1b) and (1B), we must know the displacement W_2, obtained at pressure p_2, and the value of the derivatives at any two points of the characteristic curve. It will be desirable that these should be the derivatives W_1' and W_2' at the first

$(p,0)$ and the last (p_2, W_2) points of the characteristic curve, since they are the furthest apart (Fig. 1, b and B). In this case

$$n = 1 + \frac{\lg W_1' - \lg W_2'}{\lg p_1 - \lg p_2}, \tag{3}$$

$$A = \pm \frac{W_2}{p_2^n - p_1^n}. \tag{4}$$

Here the sign + is taken for Eq. (1b) and the minus sign for Eq. (1B). For characteristic curves that are linear for the pressure (force, moment), n = 1. We consider examples of a structure of the formulas obtained, and compare the values they give for the amount of sag W with its values according to the table.

<u>Example 1</u>. The elastic element has a smoothly damped characteristic curve for pressure, designed in the interval from 800 mm Hg to 2000 mm Hg, and corresponds to the data given in the first two columns of Table 1.

p_1, mm Hg	W, mm	W, calculated from empirical formula mm	ε mm	ε %
800	0	0	0	0
900	0.260	0.276	+0.016	+6.15
1000	0.518	0.525	+0.007	+1.35
1100	0.778	0.778	0	0
1200	1.028	1.032	+0.004	+0.39
1300	1.284	1.279	-0.005	-0.39
1400	1.532	1.524	-0.008	-0.52
1500	1.784	1.765	-0.019	-0.06
1600	2.022	2.003	-0.019	-0.94
1700	2.254	2.239	-0.015	-0.67
1800	2.490	2.472	-0.018	-0.72
1900	2.711	2.703	-0.008	-0.25
2000	2.932	2.932	0	0

According to what has been said above, in a given case the relationship between <u>W</u> and <u>p</u> may be defined by Eq. (1b). On the basis of the first two columns of Table 1 we see that if the first p int (p_1 = 800 mm Hg, W_1 = 0) and the last point (p_2 = 2000 mm Hg, W_2 = 2.932 mm) of the characteristic curve, the derivatives W_1' and W_2' are:

$$W_1' = \frac{0,260 - 0}{900 - 800} = 26 \cdot 10^{-4} \ мм/мм \quad \text{Hg};$$

$$W_2' = \frac{2,932 - 2,711}{2000 - 1900} = 22,1 \cdot 10^{-4} \ мм/мм \quad \text{Hg}.$$

Substituting in Eq. (3) the numerical values for p_1, p_2, W_1', W_2' and calculating, we find that $n = 0.8226$. We further find from Eq. (4) that $A = 0.0107$. Substituting the values found in Eq. (1b) and taking into consideration that $p_1^n = 800^{0.8226} = 244.41$, we find that in the case under consideration the dependence of W on p may be defined by the following empirical equation:

$$W = 0.0107\ (p^{0.8226} - 244.41).$$

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The values for W calculated from this formula for the values of p shown in Table 1, are given in the third column of this table. The same table likewise gives the differences between the values of W as calculated by the empirical formula and the values from the table; these differences are designated by ε . The last column of Table 1 gives the values of ε expressed as percentages of the corresponding tabular values of W. It will be seen from Table 1 that the empirical formula gives values for W sufficiently close to the values of the table.

Example 2. The dependence of W on H for an elastic system intended for barometric measurement of height is given in the first two columns of Table 2.

Table 2.

H, km	W, mm	p_H, mm Hg	W, calculated from empirical formula, mm	ε mm	ε %
0	0	760	0	0	0
1	0.270	674.07	0.289	+0.019	+7.04
2	0.550	596.18	0.572	+0.022	+4.00
3	0.830	525.75	0.851	+0.021	+2.53
4	1.110	462.21	1.126	+0.016	+1.44
5	1.380	405.04	1.396	+0.016	+1.16
6	1.650	353.73	1.662	+0.012	+0.73
7	1.910	307.82	1.923	+0.013	+0.68
8	2.175	266.85	2.177	+0.002	+0.09
9	2.430	230.42	2.428	-0.002	-0.08
10	2.670	198.12	2.675	+0.005	+0.19
11	2.910	169.60	2.917	+0.007	+0.24
12	3.150	144.84	3.150	0	0

The third column gives the barometric pressures for the corresponding heights, taken from the standard atmospheric table. It will be seen from Table 2 that the elastic element in question has a smoothly dampened characteristic curve for height. Its characteristic curve for pressure corresponds to curve 2B in Fig. 1B, and is defined over the range from $p_1 = 760$ mm Hg to $p_2 = 144.84$ mm Hg.

According to what has been said, the dependence of W on p, for the elastic element under consideration, may be defined by an empirical formula of the form (1B). We see from the data of Table 2 that the derivatives W_1' and W_2' are equal at points ($p_1 = 760$ mm Hg, $W_1 = 0$) and ($p_2 = 144.84$ mm Hg,

W_2 = 3.150 mm), respectively to

$$W'_1 = \frac{0,272 - 0}{674,07 - 760} = -31,4 \cdot 10^{-4}\ мм/мм \quad \text{Hg};$$

$$W'_2 = \frac{3,150 - 2,910}{144,84 - 169,60} = -96,9 \cdot 10^{-4}\ мм/мм \quad \text{Hg}.$$

Substituting in Eq. (3) the numerical values for W_1', W_2', p_1, p_2 and carrying out the calculations, we find that n = 0.3209. Further, from Eq. F
(4) we find that A = 0.9088. Substituting the numerical values for n and 3
A in Eq. (1B) and taking into consideration the fact that p_1^n = 4
$760^{0.3209}$ = 8.402, we get the following empirical formula, provided the quantities W and p_H, for the elastic element under consideration:

$$W = 0.9088\ (8.402 - p_H^{0.3209}).$$

The value for W calculated from this formula, and likewise the values for ε , both absolute and as expressed as percentages of the tabular values for W, are given in Table 2. We see from this table that the empirical formula gives values for W that are sufficiently close to the tabular values.

We now pass to the derivation of formulas for determining the temperature flexure of an elastic element from its pressure characteristic curve, obtained for a definite temperature, which we shall designate as t_x. Furthermore, all the magnitudes pertaining to this characteristic curve will carry the index "x".

We introduce the following designations:

E_{20} - modulus of elasticity of the material of the elastic element at temperature t = + 20°C;

$m = \frac{1}{E_{20}}\frac{dE}{dt}$ - temperature coefficient of the modulus of elasticity of the elastic elements;

W - the sag of the elastic element obtained as the pressure changes from 0 to p; this sag will be called the full sag of the elastic element;

W - the sag of the elastic element obtained as the pressure changes from p_1 to p; this displacement will be called the working or useful displacement of the elastic element.

The designations E, W , W at any temperature will be provided with an index indicating that temperature. Equation (1a) gives the full displacement (in the given case, it is also the working displacement), and Equations (1b) and (1B) give the working displacement.

The full displacement of the elastic elements, from which Eq. (1b) and (1B) apply, is equal to

$$W_n = Ap_1^n \pm W = Ap^n \quad \left(A = \frac{c}{E^n}\right). \tag{5}$$

Here the + sign relates to Eq. (1b) and the - sign to Eq. (1B). The full sag in all the cases considered is equal to Ap^n.

The temperature flexure of an elastic element, obtained as the temperature goes from t_x to $\underline{t}$, for an unchanging value of the pressure $\underline{p}$, will be equal to

$$\Delta W_{t_x, t} = \pm (W_{n\,t} - W_{n.\,x}) = \pm W_{n.\,x}\left(\frac{W_{n\,t}}{W_{n.\,x}} - 1\right).$$

Here and further the + sign relates elastic elements whose characteristic curves are defined by Eq. (1a) and (1b), and the - sign corresponds to elastic elements having characteristic curves defined by Eq. (1B). On the basis of Eq. (1a) and (5), we have

$$W_{n\,t} = c\left(\frac{p}{E_t}\right)^n; \quad W_{n.\,x} = W_{n\,t_x} = c\left(\frac{p}{E_{t_x}}\right)^n.$$

Substituting these values for W_{nt}, W_{nt_x} in the parentheses of the expression for $\Delta W_{t_x, t}$, we obtain:

$$\Delta W_{t_x, t} = \pm W_{n.\,x}\left[\left(\frac{E_{t_x}}{E_t}\right)^n - 1\right].$$

Assuming that the relationship of the modulus of elasticity and the temperature is defined by the equation

$$E_t = E_{20}\,[1 - m\,(t - 20)],$$

we may write down the following expression for $\Delta W_{t_x, t}$ in the form

$$\Delta W_{t_x, t} = \pm W'_{n.\,x}\left\{\frac{[1 - m\,(t_x - 20)]^n}{[1 - m\,(t - 20)]^n} - 1\right\} \tag{6}$$

For elastic elements with a characteristic curve which is linear for pressure (force, moment), n = 1 and Eq. (6), after some elementary transformations assumes the form

$$\Delta W_{t_x, t} = \pm \frac{W_{n.\,x}\,m\,(t - t_x)}{1 - m\,(t - 20)}. \tag{7}$$

This equation is precise and makes it possible to calculate the temperature flexure of elastic elements which have a linear characteristic curve for pressure (force, moment), obtained as the temperature changes from t_x to $\underline{t}$. The magnitude W_{π_x} is taken directly from the characteristic curve obtained for temperature t_x.

We return to the general equation (6). For practically possible values for the temperatures t_x and $\underline{t}$, the derivatives $m(t_x-20)$ and $m(t-20)$ are considerably less than unity. Consequently, without great error, it may be assumed in Eq. (6) that

$$[1 - m(t_x - 20)]^n = 1 - nm(t_x - 20);$$
$$[1 - m(t - 20)]^n = 1 - nm(t - 20),$$

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after which it takes the form

$$\Delta W_{t_x, t} = \pm \frac{nm W_{\text{п. x}} (t - t_x)}{1 - nm(t - 20)}. \qquad (8)$$

For n=1, this equation is identical with Eq. (7).

It is often necessary to know the temperature flexure of an elastic element obtained as the temperature changes from a value $t_1 \neq t_x$ to a value $t_2 \neq t_x$, with unchanged pressure $\underline{p}$. We denote this flexure as $\Delta W_{t_1, t_2}$. Our aim is to determine this quantity, starting from the characteristic curve taken for temperature t_x. For this purpose we represent $\Delta W_{t_1, t_2}$ in the form

$$\Delta W_{t_1, t_2} = \Delta W_{t_x, t_2} - \Delta W_{t_x, t_1}.$$

Sags $\Delta W_{t_x, t_1}$ and $\Delta W_{t_x, t_2}$ are defined by Eq. (8). Setting $t=t_1$ in it, we obtain the expression for $\Delta W_{t_x, t_1}$. Taking $t=t_2$, we have the formula for $\Delta W_{t_x, t_2}$. Inserting in the expression for $\Delta W_{t_1, t_2}$ the values found for $\Delta W_{t_x, t_1}$ and $\Delta W_{t_x, t_2}$, we obtain

$$\Delta W_{t_1, t_2} = \pm nm W_{\text{п. x}} \left\{ \frac{t_2 - t_x}{1 - nm(t_2 - 20)} - \frac{t_1 - t_x}{1 - nm(t_1 - 20)} \right\}.$$

Reducing the expression within braces to a common denominator, we have

$$\Delta W_{t_1, t_2} = \pm nm W_{\text{п. x}} \frac{[1 - nm(t_x - 20)](t_2 - t_1)}{[1 - nm(t_1 - 20)][1 - nm(t_2 - 20)]}. \qquad (9)$$

For $t_1 = t_x$, $t_2 = t$ in this formula becomes Eq. (8). Accordingly, Eq. (9) is more general than Eq. (8). Henceforth, therefore, we shall consider only expressions for $\Delta W_{t_1, t_2}$. If the characteristic curve of a rigid element is linear for pressure (force, moment), then n=1 and Eq. (9) in that case assumes the form

$$\Delta W_{t_1,t_2} = \pm m W_{п.x} \frac{[1 - m(t_x - 20)](t_2 - t_1)}{[1 - m(t_1 - 20)][1 - m(t_2 - 20)]}. \qquad (10)$$

Since in Eq. (9) and (10) each of the quantities

$nm(t_x-20)$, $nm(t_1-20)$, $nm(t_2-20)$

$m(t_x-20)$, $m(t_1-20)$, $m(t_2-20)$

for practically possible temperature values t_x, t_1, t_2 is considerably less than zero, then, neglecting their squares and products, Eq. (9) and (10) may respectively be replaced by the following simpler expressions:

$$\Delta W_{t_1,t_2} = \pm nm W_{п.x} [1 + nm(t_1 + t_2 - t_x - 20)](t_2 - t_1); \qquad (9a)$$

$$\Delta W_{t_1,t_2} = \pm m W_{п.x} [1 + m(t_1 + t_2 - t_x - 20)](t_2 - t_1). \qquad (10a)$$

It will be seen from Eq. (9), (10), (9a) and (10a) that the temperature flexure of an elastic element is directly proportional to its full sag $W_{\pi,x}$. Consequently, in elastic elements in which the dependence of W on p is defined by Eq. (1a) or (1b), as W increases, so does $\Delta W_{t_1,t_2}$. There is a reverse picture for elastic elements where the dependence of W on p goes according to Eq. (1B); there, the flexure $\Delta W_{t_1,t_2}$ decreases as W increases. In particular, the latter is the case for the elastic elements of barometric altimeters. If they are not equipped with any temperature compensators, then as the height increases the temperature error obtained as the temperature changes by 1°C will decrease.

Thus, Eq. (9), (10), (9a), and (10a) enable us to determine, from the known characteristic curve of the full sag $W_{\pi.x} = f(p)$ of any concrete elastic element, obtained at temperature t_x, the magnitude of its temperature flexure $\Delta W_{t_1,t_2}$ for any point of the characteristic curve. It does not matter for what magnitude the characteristic was obtained - for pressure (force, moment) or for the magnitude for whose indirect measurement the elastic element in question is used. However, the characteristic curve of the complete sag is not always known, with the result that Eq. (9), (10), (9a), and (10a) are not always suitable for practical calculations. Therefore, we transform into a form that is suitable for calculating $\Delta W_{t_1,t_2}$ either from characteristic curves of the complete sag or from characteristic curves of the working (useful) sag. Assuming that the magnitudes entering into Eq. (5) are defined at t_x, we rewrite it in the form

$$W_{п,x} = A_x p^n.$$

Differentiating this expression according to p, we obtain

$$W'_{n,x} = A_x n p^{n-1} = \frac{nW_{n.x}}{p}.$$

Hence

$$nW_{n.x} = pW'_{n.x} \tag{11}$$

or, taking into consideration the fact that $W'_{n.x} = W'_x$, we have

$$nW_{n.x} = pW'_x.$$

By means of this equality Eq. (9) is transformed into the form

$$\Delta W_{t_1,t_2} = \pm \frac{mpW'_x[1-nm(t_x-20)](t_2-t_1)}{[1-nm(t_1-20)][1-nm(t_2-20)]}. \tag{12}$$

Correspondingly, Eq. (9a) assumes the form

$$\Delta W_{t_1,t_2} = \pm mpW'_x[1+nm(t_1+t_2-t_x-20)](t_2-t_1). \tag{12a}$$

To calculate by means of these formulas it is necessary to know the characteristic curve for the elastic element for pressure (force, moment); it is a matter of indifference whether for the complete or the working (useful) sag. The value of the derivative $W'_x = \frac{dW_x}{dp}$ for pressure p must take into account the sign, and may be taken as equal to the ratio of ΔW_x, the increment of the sag, to Δp, the corresponding increment in the pressure (force, moment).

If it is required to determine, by means of Eq. (12) or (12a), the temperature flexures of elastic elements prepared according to identical plans, then the nominal value of n should be taken, since the variations of this magnitude that are possible in practice with various elastic elements, even referring to different lots, cannot in this case have any essential effect on the precision with which $\Delta W_{t_1,t_2}$ are defined.

When Eq. (9) and (9a) are used, which is reasonable for elastic elements whose characteristic curves have the form represented in Fig. 1a, the value of n may be determined according to Eq. (11).

If the characteristic curve of an elastic element is linear for pressure (force, moment), then n = 1 and W'_x = const, and is equal to $\frac{W_{2x}}{p_2-p_1}$

where p_1 is the pressure corresponding to the initial point of the characteristic curve ($W_x = 0$); p_2 and W_{2x} are the pressure and sag corresponding to its final point. Hence, for characteristic curves which are linear for pressure (force, moment), we have, instead of Eq. (10) and (10a), the equations, respectively

$$\Delta W_{t_1, t_2} = \pm \frac{mpW_{2x}}{p_2 - p_1} \frac{[1 - m(t_x - 20)](t_2 - t_1)}{[1 - m(t_1 - 20)][1 - m(t_2 - 20)]} \tag{13}$$

and

$$\Delta W_{t_1, t_2} = \pm \frac{mpW_{2x}}{p_2 - p_1} [1 + m(t_1 + t_2 - t_x - 20)](t_2 - t_1). \tag{13a}$$

Equation (13), is exact like Eq. (10).

For characteristic curves defined by Eq. (1a) and represented in Fig. 1a, $p_1 = 0$.

In many cases, it is possible in Eq. (12a) and (13a) to neglect the terms $nm(t_1+t_2-t_x-20)$ and $m(t_1+t_2-t_x-20)$, which are small in comparison to unity. In these cases, we obtain, instead of Eq. (12a), the following simple equation for defining the temperature flexure of elastic elements with nonlinear characteristic curves for pressure (force, moment):

$$\Delta W_{t_1, t_2} = \pm mpW'_x (t_2 - t_1). \tag{14}$$

The relative error of this equation as compared to Eq. (12a) equals

$$\delta_{(14)} = \frac{nm(t_1 + t_2 - t_x - 20)}{1 + nm(t_1 + t_2 - t_x - 20)} \cdot 100\%. \tag{15}$$

Correspondingly, for elastic elements with linear characteristic curves for pressure (force, moment), we have instead of Eq. (13a) the equation

$$\Delta W_{t_1, t_2} = \pm \frac{mpW_{2x}}{p_2 - p_1} (t_2 - t_1), \tag{16}$$

which, as compared to Eq. (13a), gives the relative error equal to

$$\delta_{(16)} = \frac{m(t_1 + t_2 - t_x - 20)}{1 + m(t_1 + t_2 - t_x - 20)} 100\%. \tag{17}$$

The possibility of employing Eq. (14) and (16) is evaluated with the aid of Eq. (15) and (17). To this end, it is advisable to make use of the graph in Fig. 2, where the value of δ is given by the function

$$K = nm\,(t_1 + t_2 - t_x - 20).$$

By way of examples of the use of the equations that have been derived, we determine the temperature flexures of the elastic elements considered in the two previous examples. We shall assume that both elastic elements are made of phosphor bronze BrOPh 6.5-0.4, for which m = 0.00048 1/°C, and that their characteristic curves were obtained for temperature t_x = + 20°C. We determine the temperature flexure of both elastic elements at the first ($p = p_1$) and last ($p = p_2$) points of their characteristic curves, for temperature changes over the following ranges: F

1) t_1 = +20°C, t_2 = +21°C; 3 4
2) t_1 = +49°C, t_2 = +50°C;
3) t_1 = -59°C, t_2 = -60°C;
4) t_1 = +20°C, t_2 = -60°C;
5) t_1 = +20°C, t_2 = +50°C.

We make the calculation both according to Eq. (12) and Eq. (12a) and (14), which enables us to evaluate the accuracy of Eq. (12a) and (14) as compared to Eq. (12). The values of the derivatives W_x' at the initial and terminal points of the characteristic curve, as well as the values for <u>n</u> are taken as equal to the quantities previously determined for the characteristic curves under consideration (cf. Examples 1 and 2). The results of the calculations are given in Table 3, which also gives the differences between the values for W_{t_1,t_2}, calculated from Eq. (14) and (12), expressed both in millimeters and in percentages of the values for $\Delta W_{t_1,t_1}$, determined from Eq. (12). These differences are designated by δ.

It appears from Table 3 that Eq. (12) and (12a) give practically identical results. The same can be said for Eq. (13) and (13a). Accordingly, Eq. (12a) and (13a) should be used in the calculations, since they are simpler, and practically equivalent in precision to Eq. (12) and (13). It likewise appears from Table 3 that in the examples considered the precision of Eq. (14) may be considered as adequate for practical purposes. Table 3 shows that the values for the error δ for the elastic element of Example 1 is more than twice as great as the value for the elastic element considered in Example 2. The explanation for this is that the two elastic elements have different values for the product nm. In the elastic element of Example 1, nm = $3.948\cdot10^{-4}$ 1/°C, while in the elastic element of Example 2 nm = $1.54\cdot10^{-4}$ 1/°C. The smaller nm, the smaller δ .

The equations that have been derived are valid for any elastic elements whose characteristic curves are subject to any of laws (1a), (1b), (1B). It does not matter in this connection whether the elastic element is used to sense pressure <u>p</u> or force <u>P</u> or moment <u>M</u> (coil springs, spiral springs, etc.). In the latter cases, it will be necessary to replace <u>p</u> in the equations for W_{t_1,t_2} by <u>P</u> or <u>M</u>, respectively. If the elastic element operates by torsion, then <u>m</u> in the equations in question must be replaced by the temperature

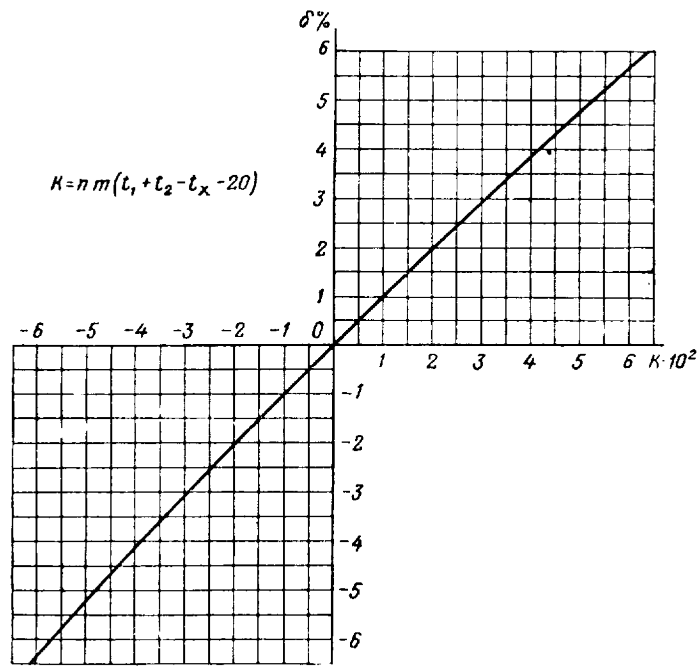

Fig. 2. Graph of relative error δ

$$\frac{cp^n}{E_{20}^n[1-m(t_x-20)]^n} = W_{n.x},$$

we find

$$W_{nt} = W_{n.x} + \frac{nmW_{n.x}(t-t_x)}{1-m(t_x-20)}.$$

Hence

$$\Delta W_{t_x,t} = W_{nt} - W_{n.x} = \frac{nmW_{n.x}(t-t_x)}{1-m(t_x-20)}.$$ F 3 4

The term $m(t_x-20)$ is small compared to unity. Neglecting it and employing Eq. (11), we have

$$\Delta W_{t_x,t} = mpW'_x(t-t_x). \tag{18}$$

As previously, we represent $\Delta W_{t_1,t_2}$, the temperature flexure of the elastic element, which occurs as the temperature changes from $t_1 \neq t_x$ to $t_2 \neq t_x$, the pressure p being unchanged, in the form

$$\Delta W_{t_1,t_2} = \Delta W_{t_x,t_2} - \Delta W_{t_x,}$$

If we now substitute for the flexures $\Delta W_{t_x,t_1}$ and $\Delta W_{t_x,t_2}$ the values of them determined by Eq. (18), we have

$$\Delta W_{t_1,t_2} = mpW'_x(t_2-t_1)$$

This equation coincides exactly with Eq. (14). Since Eq. (18) was derived on the assumption that the difference $(t-t_x)$ is small, then the present equation for $\Delta W_{t_1,t_2}$ is valid only for t_1 and t_2 in the neighborhood of t_x. This applies completely to Eq. (16) as well, which is only a particular case of Eq. (14). In this way, Eq. (14) and (16) are capable of supplying a precise result only in those cases when t_1 and t_2 are close to t_x.

Up to the present we have considered elastic elements whose characteristic curves are monotonous functions in the strict sense and have no singular points. We now go over to considering the temperature flexures of elastic elements which have characteristic curves for the force P (pressure, moment) that are made up of broken line segments. In this we confine ourself to the case in which the characteristic curve has a single point of inflection (Fig. 3). As an example of such an elastic element we may take an elastic element consisting of two compression springs, schematically represented in Fig. 4. We assume that the transverse moduli and their temperature coefficients are the same for both springs.

Although the section OA (see Fig. 3) of spring 1 is operative (see Fig. 4), so that calculation of the temperature flexure for this section of the characteristic curve should be conducted according to the equations derived above. From the discontinuity point A (P_{disc} W_{disc}) both springs are operative. In this case the force P and the sag W are connected by the relationship

$$P=a_1GW+a_2G(W-W_{disc}), \qquad (19)$$

where a_1, a_2 - constant coefficients depending on the dimensions and the number of turns of the 1st and the 2nd spring respectively:
G - transverse modulus.

From this expression we find that the section AB (see Fig. 3)

$$W=\frac{P}{G(a_1+a_2)}+\frac{a_2W_{disc}}{a_1+a_2}. \qquad (20)$$

This relationship between W and P coincides with the relationship defined by Eq. (1b). Hence, the equations previously derived for the temperature flexure of elastic elements are applicable as well to the section AB of

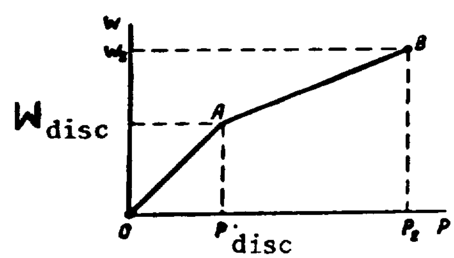

Fig. 3. Characteristic curve of an elastic element.

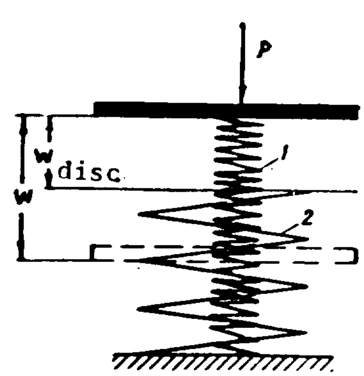

Fig. 4. Diagram of an elastic element.

the characteristic curve, now under consideration. We prove this. We consider the temperature flexure of the elastic element over the section AB. The temperature flexure obtained as the temperature varies from t_x to t, with the value of force P unchanged, will be

$$\Delta W_{t_x,\,t} = W_t - W_x .$$

Substituting for W_t and W_x in this equation the values as obtained from Eq. (20), we have

$$\Delta W_{t_x,\,t} = \frac{P}{G_t(a_1+a_2)} + \frac{a_2 W_{disc}}{a_1+a_2} - \frac{P}{G_{t_x}(a_1+a_2)} - \frac{a_2 W_{disc}}{a_1+a_2}$$ F 3 4

Hence, after some simple transformations we have

$$\Delta W_{t_x,\,t} = \frac{P}{G_{t_x}(a_1+a_2)}\left(\frac{G_{t_x}}{G_t} - 1\right).$$

For point B of the characteristic curve (see Fig. 3), Eq. (19) has the form

$$P_2 = a_1 G_{t_x} W_{2x} + a_2 G_{t_x}\left(W_{2x} - W_{disc\ x}\right)$$

Subtracting from this term by term Eq. (19), derived for point A of the characteristic curve, we have

$$G_{t_x}(a_1+a_2) = \frac{P_2 - P_{disc\ x}}{W_{2x} - W_{disc\ x}}$$

Substituting this value for $G_{t_x}(a_1 + a_2)$ in the last expression for $\Delta W_{t_x,t}$ and replacing G_{t_x} and G_t by their values, we have

$$\Delta W_{t_x,\,t} = \frac{P(W_{2x} - W_{disc\ x})}{P_2 - P_{disc\ x}}\left\{\frac{G_{20}[1 - m_1(t_x - 20)]}{G_{20}[1 - m_1(t - 20)]} - 1\right\}.$$

Reducing the expression in braces to a common denominator we finally get

$$\Delta W_{t_x,\,t} = \frac{P(W_{2x} - W_{disc\ x})}{P_2 - P_{disc\ x}}\,\frac{m_1(t - t_x)}{1 - m_1(\ - 20)}. \qquad (21)$$

We now define $\Delta W_{t_1,t_2}$, the temperature flexure of the elastic element over section AB (see Fig. 3), resulting from a temperature change from $t_1 \neq t_x$ to $t_2 \neq t_x$, with the value of force P unchanged.

As previously,

$$\Delta W_{t_1,\,t_2} = \Delta W_{t_x,\,t_2} - \Delta W_{t_x,\,t_1}.$$

Inserting in Eq. (21) first $t = t_1$ and then again $t = t_2$, we have corresponding expressions for $\Delta W_{t_x,\,t_1}$ and $\Delta W_{t_x,\,t_2}$. Inserting these in the right side of the last equation and reducing the expression obtained to a common denominator, we have

$$\Delta W_{t_1,\,t_2} = \frac{m_1 P (W_{2x} - W_{disc\,x})}{P_2 - P_{disc\,x}} \frac{[1 - m_1 (t_x - 20)]\,(t_2 - t_1)}{[1 - m_1 (t_1 - 20)]\,[1 - m_1 (t_2 - 20)]}. \qquad (22)$$

This expression, for the same reasons as given for Eq. (10), may be replaced by the following simpler equation:

$$\Delta W_{t_1,\,t_2} = \frac{m_1 P (W_{2x} - W_{disc\,x})}{P_2 - P_{disc\,x}} [1 + m_1 (t_1 + t_2 - t_x - 20)]\,(t_2 - t_1), \qquad (22a)$$

which is the one that should be used for determining $\Delta W_{t_1,\,t_2}$ over section AB of the characteristic curve (see Fig. 3) of the elastic element under consideration; in this the percentage error obtained is

$$\delta_{(22a)} = \frac{m_1 (t_1 + t_2 - t_x - 20)}{1 + m_1 (t_1 + t_2 - t_x - 20)}\, 100\,\%. \qquad (23)$$

The calculation according to Eq. (22a) may be replaced by calculation from the following very simple equation:

$$\Delta W_{t_1,\,t_2} = \frac{m_1 P (W_{2x} - W_{disc\,x})}{P_2 - P_{disc\,x}} (t_2 - t_1). \qquad (24)$$

We note that Eq. (22), (22a), (24) are in complete correspondence with Eq. (13), (13a) and (16).

Thus, in all the cases considered the temperature flexure at the point under consideration of the characteristic curve is equal to

$$\Delta W_{t_1,\,t_2} = m p W'_x [1 + nm (t_1 + t_2 - t_x - 20)].$$

If the force acting is concentrated force P or moment M, p must be replaced by P or M, respectively. For elastic elements operating by torsion, m_1 should be taken instead of m.

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